TypOS: An “Operating System” for Typechecking Actors

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A domain-specific language for typecheckers

An experiment in how to write typecheckers that make (more) sense.

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Concrete motivation: implementing a type theory with rich equational theory for free monoids and free Abelian groups.

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Ruling out design errors by construction: a first-order representation means we can do static analysis on the typecheckers themselves.

A Tour of TypOS

Syntax descriptions

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- atoms 'a
- cons lists [t_0 t_1 $\ldots$ t_n]
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There is a syntax description of syntax descriptions, which we use to check syntax descriptions.

Judgement forms as interaction protocols

We recast the notion of $\boxed{\text{judgement form}}$ as communication protocol:

- What to communicate (of what syntax description)?
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A basic form of session types [Honda 1993].

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A basic form of session types [Honda 1993].

For example:

```
type : ?'Type.
check : ?'Type. ?'Check.
synth : ?'Synth. !'Type.
```

Typing rules as actors

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That is: typing rules are implemented by actors, which

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Typechecking process *actor* with parent channel p is defined by

```
actor@p = ...
```

Actor constructs: winning

␣

a successful, finished actor

(Victory is silent.)

Actor constructs: failing

`# "error message"`

an unsuccessful, finished actor

Actor constructs: printing

```
PRINTF "message text".
```

printing a message before continuing

Actor constructs: generating fresh meta variables

$$sd ? X \, .$$

generate a fresh meta X of syntax description sd

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Meta variables stand for *unknown* terms.

Actor constructs: matching on terms

$$\texttt{case t} \left\{\ p_1\ \texttt{->}\ a_1\ \texttt{;}\ \texttt{...}\right\}$$

match term t against patterns p_i; continue as actor a_i when matching

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$$\texttt{case t } \{ \; p_1 \; \texttt{->} \; a_1 \; \texttt{;} \; \texttt{...} \}$$

match term t against patterns p_i; continue as actor a_i when matching

Blocks if t is a metavariable.

Actor constructs: forking

```
a | b
```

continue as `a` and `b` in parallel

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continue as `a` and `b` in parallel

Progress in `b` might enable further progress in `a` and vice versa.

Actor constructs: declaring constraints

$$t_1 \sim t_2$$

make t_1 unify with t_2

Actor constructs: spawning children

$$actor@p.$$

spawn a new child *actor* on channel p

Actor constructs: sending and receiving messages

$$p\,!\,t.$$

send term t on channel p

Actor constructs: sending and receiving messages

$$p\,!\,t.$$

send term t on channel p

$$p?t.$$

receive term t on channel p

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$$p\,!\,t.$$

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receive term t on channel p

Messages must conform to p's protocol.

Actor constructs: binding local variables

$$\backslash\, x.$$

bring fresh object variable x into scope

Actor constructs: extending local contexts

$$ctx \;\; |\text{-} \;\; x \;\; \text{->} \;\; t$$

extend declared context *ctx* to map object variable *x* to term *t*

Actor constructs: querying local contexts

$$\texttt{if}\ x\ \texttt{in}\ ctx\ \{\ t\ \texttt{->}\ a\ \}\ \texttt{else}\ b$$

Look up variable x in declared context ctx;
if found, bind associated value as t and continue as a,
otherwise continue as b

Actors for bidirectional type checking of STLC

```
check@p = p?ty. p?tm. case tm
  { ['Lam \x. body] -> 'Type?S. 'Type?T.
     ( ty ~ ['Arr S T]
     | \x. ctxt |- x -> S. check@q. q!T. q!body.)
  ; ['Emb e] -> synth@q. q!e. q?S. S ~ ty }

synth@p = p?tm. if tm in ctxt
  { S -> p!S. }
  else case tm
  { ['Ann t T] -> ( type@q. q!T.
                  | check@r. r!T. r!t.
                  | p!T. )
  ; ['App f s] -> 'Type?S. 'Type?T. p!T.
     ( synth@q. q!f. q?F. F ~ ['Arr S T]
     | check@r. r!S. r!s.) }
```

Executing actors

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We run each actor until it blocks, and then try the next one, until execution stabilises.

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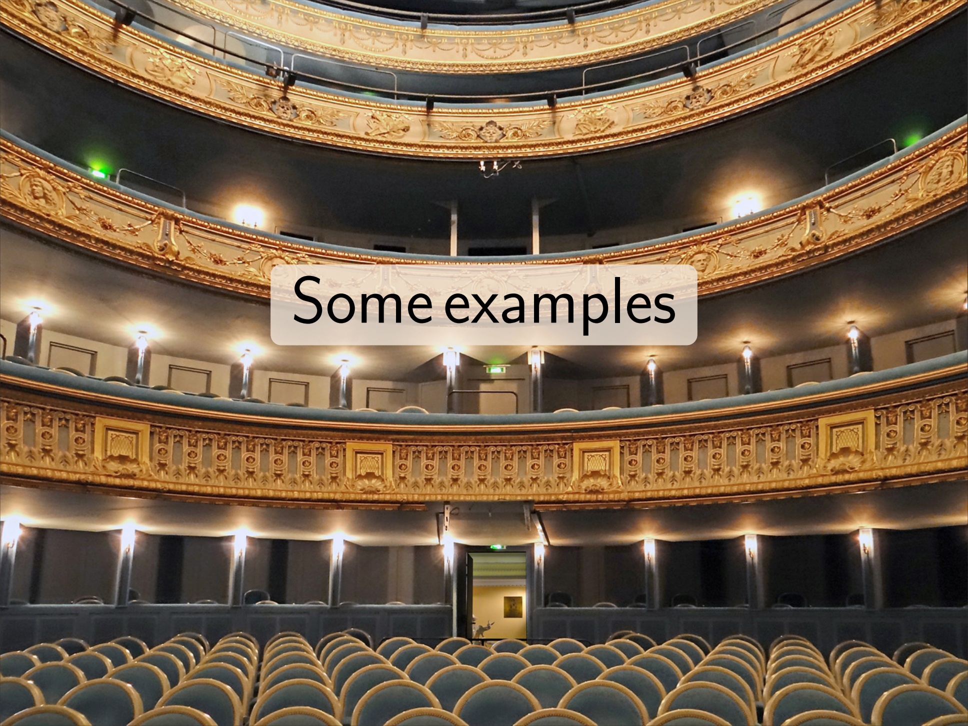
We can extract a typing derivation from the final configuration of the stack.

Some examples



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  }{z_0 : \mathbb{N} \vdash}
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\overline{\mathbb{N} \to \mathbb{N} \ni \lambda z.\underline{(\lambda\_.[\mathsf{Succ}\ \mathsf{Zero}] : \mathbb{N} \to \mathbb{N})\underline{z}}}
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typos --latex-animated=stlc-ann.tex stlc.act

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{TYPE}\ \mathbb{N} \to \mathbb{N}^{\checkmark}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathbb{N} \ni \mathsf{Zero}^{\checkmark}}}{\mathbb{N} \ni [\mathsf{Succ\ Zero}]^{\checkmark}}}{w_1 : \mathbb{N} \vdash}}{\mathbb{N} \to \mathbb{N} \ni \lambda\_.[\mathsf{Succ\ Zero}]^{\checkmark}}}{(\lambda\_.[\mathsf{Succ\ Zero}] : \mathbb{N} \to \mathbb{N}) \in \mathbb{N} \to \mathbb{N}^{\checkmark}} \qquad \dfrac{}{\mathbb{N} \ni \underline{z_0}}}{(\lambda\_.[\mathsf{Succ\ Zero}] : \mathbb{N} \to \mathbb{N})\underline{z_0} \in \mathbb{N}}}{\mathbb{N} \ni \underline{(\lambda\_.[\mathsf{Succ\ Zero}] : \mathbb{N} \to \mathbb{N})\underline{z_0}}}}{z_0 : \mathbb{N} \vdash}}{\mathbb{N} \to \mathbb{N} \ni \lambda z.\underline{(\lambda\_.[\mathsf{Succ\ Zero}] : \mathbb{N} \to \mathbb{N})\underline{z}}}
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typos --latex-animated=stlc-ann.tex stlc.act
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`typos --latex-animated=stlc-ann.tex stlc.act`

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    z_0 : \mathbb{N} \vdash \\ \hline
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typos --latex-animated=stlc-ann.tex stlc.act

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Verification of actors

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Summary and future work

TypOS is an domain-specific language for writing typecheckers.

Judgements have modes (input/output protocols), typing rules are actors (spawning and communicating with children).

A wide range of typechecking, evaluation and elaboration processes can be implemented this way.

In the future: a truly concurrent runtime.

`https://github.com/msp-strath/TypOS`

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Thank you!

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References

In order of appearance

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